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ADVANCES IN APPLYING THE CARDINALITY-CONSTRAINED MODEL FOR THE NURSE-TO-PATIENT ASSIGNMENT PROBLEM IN HOME CARE

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Abstract

In this paper we discuss issues related to uncertainty in input data in a relevant health care management problem, the nurse-to-patient assignment problem under continuity of care in home care services, in which patient demands are uncertain parameters and continuity of care is pursued. We consider two variants of the problem: the first variant aims at minimizing the overtime cost, and the second variant aims at obtaining a fair workload among nurses. In the first variant different possible levels of demand are taken into account. In the second variant we investigate the behavior of different objective functions in ensuring a fair workload. We present cardinality-constrained robust formulations for both problems. For the maximum fairness case, some results on a toy instance are given.

1. Introduction

Randomness in data and parameters is a common feature of several optimization problems, and arises in many applications, spreading from telecommunications, where uncertainty is usually associated to traffic demands, to health care, where uncertainty is related to patients' conditions. Indeed, uncertainty is inherent in many health care optimization problems and cannot be neglected, as it may have a significant impact on the problem solution. In locating emergency vehicles, uncertainty is associated to the availability of ambulances (probabilistic models for the ambulance location problem are reported in [6]), whereas uncertainty is related to the duration of surgery in planning and scheduling operating room theaters (see [13]). Uncertainty also occurs in managing Home Care (HC) services. HC services consist of providing cares to patients at their domicile rather than in hospital. It allows both to reduce hospitalization costs and to improve patients' quality of life. Managing human resources in HC services is a difficult task, which is made even more complex by uncertain patients' demands. HC providers must synchronize the use of the resources at the patient's domicile, while usually delivering the service to a large number of patients in a vast territory. Furthermore, random events affect the service delivery, undermine the feasibility of plans, and cause a high variability in the workload charged to nurses and, consequently, in the cost of the service provided. One of the most critical and frequent of such events is a sudden variation in the amount of service required by patients, which is in general highly variable. In addition, several providers pursue the continuity of care. This means that the HC provider assigns only one nurse to each patient, the *reference nurse*, and the assignment is kept for a long period. Continuity is an important quality indicator of the provided service, because the patient receives care from the same nurse instead of continuously developing new relationships, and potential loss of information among operators is avoided. However, continuity of care constraint limits the flexibility of the service; thus, for a good trade-off between quality and flexibility, continuity of care should be preserved at least for critical patients or patients with particular needs.

In this paper, we focus on the nurse-to-patient assignment problem under continuity of care, which requires assigning each newly admitted patient to his/her reference nurse, under the appropriate continuity of care requirement. Different approaches have been applied to solve the problem while taking into account patients' demand variability. Among the others, this assignment problem has been solved applying the cardinality-constrained approach [9]. Such approach provides good quality solutions in a reasonable computational time, showing to be an effective tool for the considered problem. However, some improvements are possible and in this report we propose two different directions of improvement.

The report is structured as follows. A literature review of the problems that arise in planning HC services is reported in Section 2, while the description of the problem addressed in this paper is given in Section 3. The directions of improvement are then presented in Section 4, and detailed in Sections 5 and 6, respectively. Final discussions and conclusion are reported in Section 7.

2. State of the art

HC management involves several resources and must take into account many requirements and constraints. It is related to nurse rostering in hospitals (see [11, 7]) as it deals with nurse management. However, HC management involves several issues which are not usually addressed in nurse rostering problems, such as the continuity of care [14] or the burnout risk [5], which make the HC nurse management peculiar. The main issues to be considered in HC resource planning are the partitioning of a territory into a given number of districts, the dimensioning of human resources, the assignment of visits to operators (or patients to operators in the case of continuity of care), the scheduling of nurses' duties and the routing optimization. Literature papers about HC can be mainly divided into two groups: a first group dealing with daily schedule of visits and routing of nurses, and a second group dealing with staff planning and management in a mid-term and long-term perspective. From a long-term point of view, the districting problem consists of grouping patients and nurses according to geographical and skill compatibility [3, 18]. The dimensioning of human resources consists of determining the number of operators, together with their skills, to meet patients' demand in each district. Uncertainty in the demands may be taken into account [12], and funding is to be taken into account in dimensioning HC resources, as well [8].

The nurse-to-patient assignment problem under continuity of care is part of the mid-term management. The uncertainty in patients' demands can be considered in assigning the visits. According to the continuity of care, patients (and not single visits) are assigned to nurses, and each patient is assigned to the same nurse for the whole length of the treatment. Thus, the problem consists of assigning nurses to patients in a fair way. It has been rarely studied as a stand alone problem, i.e., not considering the scheduling [4], and, to the best of our knowledge, the assignment problem taking into account the continuity of care issue is only marginally addressed in the literature [5, 15, 19, 20]. Besides, continuity of care is often considered as an objective rather than a strict requirement and, therefore, dealt with as a soft-constraint rather than as a hard one (see for instance [24]). If continuity of care is not considered, the assignment problem turns out to be an assignment of operators to visits, with the aim of jointly optimizing the assignment of operators to visits and the scheduling and routing problem (see [26, 25]). As mentioned, uncertainty inherently arises in HC due to unpredictable changes in patients' needs. It affects the personnel workload and the number of patients who can be treated. Different approaches are usually applied to deal with uncertainty in health care problems, such as probabilistic models or stochastic optimization approaches. However, the nurse-to-patients assignment problem in which both continuity of care and demand uncertainty are considered has been rarely addressed in the literature. In [17] uncertainty is managed by representing the whole system as a Markov chain and developing admittance policies for patients. The problem was tackled with the stochastic programming on one side [20], and with analytical policies on the other [19]. However, both these approaches proved to be limited, even if they improved the quality of the assignment upon those provided by the HC structures in the practice. Indeed, the stochastic programming approach is based on scenario generation and, due to the high number of patients and the associated demand variability, should include a very high number of scenarios. Only a limited number of them can be consequently considered for a computationally feasible solution and, therefore, a high expected value of perfect information (EVPI) and a low value of the stochastic solution (VSS) are obtained [20]. The analytical policies are related to strict assumptions regarding the shape of workload probability density functions, the number of assignable patients (one patient at a time after deciding an ordering of new patients) and the number of periods in the planning horizon, which is equal to one [19].

Recently, the cardinality-constrained model has been applied for solving the nurse-to-patient assignment problem under continuity of care in HC [9]. This is an innovative robust optimization approach, which has been proposed in [2] and has been already applied in several fields, spreading from portfolio optimization to telecommunication network design. The approach allows to account for a certain degree of uncertainty with a reasonable computational effort, providing a trade-off between computational time and robustness. In addition, it can be tuned to take into account the specific degree of risk the decision maker accepts. Although the approach seems to fit well to many health care problems, and in particular to the HC planning, to the best of our knowledge it has not be applied to many health care problems until now. Besides [9], there are the only five papers (i.e., [1, 13, 16, 10, 22]) found in September 2013 through a search on ISI Web of Knowledge and Scopus among the papers citing [2] and referring to Health Care. However, none deals with HC management.

3. Problem description

The nurse-to-patient assignment problem consists of assigning a set of HC patients P to a set of nurses I over a time horizon T represented by a set of time slots t. Continuity of care is taken into account; indeed, different continuity of care requirements are considered, depending on the type of patient and on his/her requests. Hence, the set of patients P is partitioned into five subsets:

1. **Patients who require hard continuity of care (C)**

   P_c^a: set of patients who require hard continuity of care – i.e., their reference nurse cannot be changed – and are already under treatment (and therefore assigned) at the beginning of the time horizon. As they cannot be reassigned, they keep their reference nurse.

   P_c^n: set of patients who require hard continuity of care and start their treatment during the time horizon (therefore, they are not yet assigned).

2. **Patients who require partial continuity of care (PC)**

   P_{pc}^a: set of patients who require partial continuity of care – i.e., their reference nurse can be changed although it is preferable not to – and are already under treatment (and therefore assigned) at the beginning of the time horizon.

   P_{pc}^n: set of patients who require partial continuity of care and start their treatment during the time horizon (therefore, they are not yet assigned). Similarly to patients in P_{pc}^a, they can be reassigned at the beginning of each time slot t with a reassignment cost γ.

3. **Patients who do not require continuity of care (NC)**

   P_{nc}: set of patients who do not require continuity of care. They can be assigned to more than one nurse even in the same time slot t and the assignments can be changed from a time slot to another without reassignment costs.

The division in districts (usually based on territory and skills) is taken into account, i.e., each district is assigned to a subset of nurses. A parameter m_{ij} is given for each nurse $i \in I$ and patient $j \in P$, which is equal to 1 if i operates in the district of j, and 0 otherwise. The amount of working time required by each patient $j \in P$ during each time slot $t \in T$ is an uncertain parameter r_{jt}. Each nurse $i \in I$ has an amount of available working time v_i in each time slot t. If the workload amount of a nurse exceeds the available time, the overtime must be paid to the nurse with a cost that varies according to the overtime amount. A limit is also given to the total workload amount of each nurse $i \in I$, which cannot exceeds twice the value of v_i. The formulation of the problem developed in [9] is described in Section 3.1.

3.1. Cardinality-constrained formulation for cost reduction

The objective aims at minimizing a sum of nurses' overtimes cost and reassignments penalties of patients with partial continuity of care. For describing the overtime costs, a set of overtime levels L_i is defined for each nurse $i \in I$ and two parameters are given for each level $l \in L_i$: a threshold Δ_i^l and a cost per time unit c_l (c_l is the cost for each overtime unit above $v_i + \sum_{k=1}^{l-1} \Delta_i^k$ and below $v_i + \sum_{k=1}^{l} \Delta_i^k$). Costs c_l are such that $c_l < c_{l+1}$ to get a monotonically increasing stepwise function. For the minimization of the reassignments, a binary variable y_j^t is introduced for each patient $j \in P_{pc}^a \cup P_{pc}^n$: y_j^t is equal to 1 if the assignment of j is changed from $t-1$ to t, and 0 otherwise. The following decision variables are used to model the assignments. A binary variable x_{ji} is defined for each patient $j \in P_c^a \cup P_c^n$ and each nurse $i \in I$: x_{ji} is equal to 1 if j is assigned to i during the whole time horizon, and 0 otherwise. A binary variable ξ_{ji}^t is defined for each patient $j \in P_{pc}^a \cup P_{pc}^n$ who requires partial continuity of care, each nurse $i \in I$ and each time slot $t \in T$: ξ_{ji}^t is equal to 1 if i is in charge of j during t, and 0 otherwise. The assignments of patients to reference nurses before the beginning of the considered time horizon are described by parameters $\tilde{x}_{ji}$ for each patient $j \in P_c^a \cup P_{pc}^a$ and each nurse $i \in I$: $\tilde{x}_{ji}$ is equal to 1 if j is initially assigned to i, and 0 otherwise. The fraction of the time needed by patient $j \in P_{nc}$ during time slot $t \in T$, which is provided by nurse $i \in I$, is represented by a continuous variable $\chi_{ji}^t \in [0, 1]$. The overtime amount assigned to each nurse $i \in I$ during each time slot $t \in T$ is described by a continuous variable w_{it}^l for each level $l \in L_i$, which represents the overtime related to cost c_l. The model is thus written as follows [9]:

$$\min \left\{ \sum_{i \in I} \sum_{t \in T} \sum_{l \in L_i} c_l w_{it}^l + \gamma \sum_{j \in P_{pc}^a \cup P_{pc}^n} \sum_{t \in T} y_j^t \right\}. \tag{1}$$

s.t.

$$\sum_{i \in I} m_{ij} x_{ji} = 1, \qquad \forall j \in P_c^a \cup P_c^n \tag{2}$$

$$\sum_{i \in I} m_{ij} \xi_{ji}^t = 1, \qquad \forall j \in P_{pc}^a \cup P_{pc}^n, t \in T \tag{3}$$

$$\sum_{i \in I} m_{ij} \chi_{ji}^t = 1, \qquad \forall j \in P_{nc}, t \in T \tag{4}$$

$$\begin{aligned} &\sum_{j \in P_c^a \cup P_c^n} r_{jt} x_{ji} + \sum_{j \in P_{pc}^a \cup P_{pc}^n} r_{jt} \xi_{ji}^t + \\ &\quad + \sum_{j \in P_{nc}} r_{jt} \chi_{ji}^t \le v_i + \sum_{l \in L_i} w_{it}^l, \qquad \forall i \in I, t \in T \end{aligned} \tag{5}$$

$$0 \le w_{it}^l \le \Delta_i^l, \qquad \forall i \in I, t \in T, l \in L_i \tag{6}$$

$$x_{ji} \ge \tilde{x}_{ji}, \qquad \forall i \in I, j \in P_c^a \tag{7}$$

$$
\begin{aligned}
y_j^t &\geq \xi_{ji}^t - \xi_{ji}^{t-1}, && \forall t \in T \setminus \{t_1\}, j \in P_{pc}^a \cup P_{pc}^n, i \in I && (8)\\
y_j^{t_1} &\geq \xi_{ji}^{t_1} - \tilde{x}_{ji}, && \forall j \in P_{pc}^a, i \in I && (9)\\
x_{ji} &\in \{0,1\}, && \forall j \in P_c^a \cup P_c^n, i \in I && (10)\\
\xi_{ji}^t &\in \{0,1\}, && \forall j \in P_{pc}^a \cup P_{pc}^n, t \in T, i \in I && (11)\\
\chi_{ji}^t &\in [0,1], && \forall j \in P_{nc}, t \in T, i \in I && (12)\\
y_j^t &\in \{0,1\}, && \forall j \in P_{nc}, t \in T && (13)
\end{aligned}
$$

Constraints (2)–(4) guarantee that each patient is assigned to a suitable nurse, taking into account the district compatibility and the pertinent continuity of care requirement. Constraints (5) compute the nurse workload (left hand side) and the overtime, which is divided into the different cost levels (right hand side). Constraints (6) set the thresholds for the overtime workload of each level. Constraints (7) guarantee that patients belonging to the subset P_c^a do not change their assignment at the beginning of the considered time horizon. Finally, constraints (8) and (9) compute the number of reassignments. In particular, constraints (9) compute the number of reassignments at the initial time slot t_1 for patients in the subset P_{pc}^a. The robustness is inserted in this formulation according to the standard cardinality- constrained approach [2].

- The stochastic amount of working time r_{jt} is modeled considering an expected value $\bar{r}_{jt}$ and a maximum value $\bar{r}_{jt} + \hat{r}_{jt}$.
- Three sets S_c^{it}, S_{pc}^{it} and S_{nc}^{it} are introduced for each nurse $i \in I$ and each time slot $t \in T$. $S_c^{it} \subseteq P_c^a \cup P_c^n$ is the subset of patients requiring hard continuity of care and assigned to i, whose demand charged to nurse i in time slot t is equal to the maximum treatment time (i.e., $\bar{r}_{jt} + \hat{r}_{jt}$). $S_{pc}^{it} \subseteq P_{pc}^a \cup P_{pc}^n$ and $S_{nc}^{it} \subseteq P_{nc}$ are analogously defined for patients requiring partial and no continuity of care, respectively.
- Three cardinality parameters Γ_c^i, Γ_{pc}^i and Γ_{nc}^i are introduced for each nurse $i \in I$. At most $\lfloor \Gamma_c^i \rfloor$ and $\lfloor \Gamma_{pc}^i \rfloor$ and $\lfloor \Gamma_{nc}^i \rfloor$ patients (with hard, partial and no continuity of care) are assumed to belong to these subsets, respectively.
- Further, in case Γ_c^i and Γ_{pc}^i and Γ_{nc}^i are not integer, three patients are selected for each nurse i and each time slot t, whose demand charged to i is between $\bar{r}_{jt}$ and $\bar{r}_{jt} + \hat{r}_{jt}$. We denote p_c^{it}, p_{pc}^{it} and p_{nc}^{it} such patients: p_c^{it} is a patient belonging to $P_c^a \cup P_c^n$ but not to S_c^{it}, p_{pc}^{it} is a patient belonging to $P_{pc}^a \cup P_{pc}^n$ but not to S_{pc}^{it}, and p_{nc}^{it} is a patient belonging to P_{nc} but not to S_{nc}^{it}.
- The charged demand of the other patients not belonging to S_c^{it}, S_{pc}^{it} and S_{nc}^{it} and different from p_c^{it}, p_{pc}^{it} and p_{nc}^{it} is the expected treatment time (i.e., $\bar{r}_{jt}$).

The robustness is taken into account considering the worst possible charge for each nurse i in each time slot t, given cardinalities Γ_c^i, Γ_{pc}^i and Γ_{nc}^i. Hence, the model is modified in order to include such worst case in constraints (5):

$$
\sum_{j \in P_c^a \cup P_c^n} \bar{r}_{jt} x_{ji} + \sum_{j \in P_{pc}^a \cup P_{pc}^n} \bar{r}_{jt} \xi_{ji}^t + \sum_{j \in P_{nc}} \bar{r}_{jt} \chi_{ji}^t +
$$

$$
+ \max_{\substack{S_c^{it} \cup \{p_c^{it}\} \mid S_c^{it} \subseteq P_c^a \cup P_c^n, \\ |S_c^{it}| = \lfloor \Gamma_c^i \rfloor,\, p_c^{it} \in P_c^a \cup P_c^n \setminus S_c^{it}}} \left\{ \sum_{j \in S_c^{it}} \hat{r}_{jt} x_{ji} + \left(\Gamma_c^i - \lfloor \Gamma_c^i \rfloor \right) \hat{r}_{p_c^{it} t} x_{p_c^{it} i} \right\} +
$$

$$+ \max_{\substack{S_{pc}^{it}\cup\{p_{pc}^{it}\}|\\ S_{pc}^{it}\subseteq P_{pc}^{a}\cup P_{pc}^{n}, |S_{pc}^{it}|=\lfloor\Gamma_{pc}^{i}\rfloor,\\ p_{pc}^{it}\in P_{pc}^{a}\cup P_{pc}^{n}\setminus S_{pc}^{it}}} \left\{ \sum_{j\in S_{pc}^{it}} \hat{r}_{jt}\xi_{ji}^{t} + \left(\Gamma_{pc}^{i} - \lfloor\Gamma_{pc}^{i}\rfloor\right) \hat{r}_{p_{pc}^{it}t}\xi_{p_{pc}^{it}i}^{t} \right\} + \tag{14}$$

$$+ \max_{\substack{S_{nc}^{it}\cup\{p_{nc}^{it}\}|S_{nc}^{it}\subseteq P_{nc},\\ |S_{nc}^{it}|=\lfloor\Gamma_{nc}^{i}\rfloor, p_{nc}^{it}\in P_{nc}\setminus S_{nc}^{it}}} \left\{ \sum_{j\in S_{nc}^{it}} \hat{r}_{jt}\chi_{ji}^{t} + \left(\Gamma_{nc}^{i} - \lfloor\Gamma_{nc}^{i}\rfloor\right) \hat{r}_{p_{nc}^{it}t}\chi_{p_{nc}^{it}i}^{t} \right\} \leq$$

$$\leq v_i + \sum_{l\in L_i} w_{it}^{l}, \quad \forall i \in I, t \in T$$

Let us denote with $\beta_c^{it}(x^*, \Gamma_c^i, t)$, $\beta_{pc}^{it}(\xi^*, \Gamma_{pc}^i, t)$ and $\beta_{nc}^{it}(\chi^*, \Gamma_{nc}^i, t)$ the three maxima included in (14), which are related to a given solution $\{x^*, \xi^*, \chi^*\}$. As example, $\beta_c^{it}(x^*, \Gamma_c^i, t)$ is expressed as follows:

$$\beta_c^{it}(x^*, \Gamma_c^i, t) = \max_{\substack{S_c^{it}\cup\{p_c^{it}\}|S_c^{it}\subseteq P_c^a\cup P_c^n,\\ |S_c^{it}|=\lfloor\Gamma_c^i\rfloor, p_c^{it}\in P_c^a\cup P_c^n\setminus S_c^{it}}} \left\{ \sum_{j\in S_c^{it}} \hat{r}_{jt}x_{ji}^{*} + \left(\Gamma_c^i - \lfloor\Gamma_c^i\rfloor\right) \hat{r}_{p_c^{it}t}x_{p_c^{it}i}^{*} \right\} \tag{15}$$

This is computed for each nurse $i \in I$ and each time slot $t \in T$ by solving the following linear programming problem:

$$(\mathcal{P}_c^{\beta it}) = \max \sum_{j\in P_c^a\cup P_c^n} \hat{r}_{jt}x_{ji}^{*}z_{ji}^{t} \tag{16}$$

$$\sum_{j\in P_c^a\cup P_c^n} z_{ji}^{t} \leq \Gamma_c^i \tag{17}$$

$$0 \leq z_{ji}^{t} \leq 1, \qquad \forall j \in P_c^a \cup P_c^n \tag{18}$$

where i and t are fixed, and $z_{ji}^t \in [0, 1]$ are continuous variables which represent the choice of the elements in subset S_c^{it} and the choice of p_c^{it}.[1] Let us denote with ζ_{it}^c the dual variables associated to (17) and with π_{jit}^c the dual variables associated to $z_{ji}^t \leq 1$ (18). The dual problem is written as:

$$(\mathcal{D}_c^{\beta it}) = \min \quad \Gamma_c^i\zeta_{it}^c + \sum_{j\in P_c^a\cup P_c^n} \pi_{jit}^c \tag{19}$$

$$\zeta_{it}^c + \pi_{jit}^c \geq \hat{r}_{jt}x_{ji}^{*}, \qquad \forall j \in P_c^a \cup P_c^n \tag{20}$$

$$\pi_{jit}^c \geq 0, \qquad \forall j \in P_c^a \cup P_c^n \tag{21}$$

$$\zeta_{it}^c \geq 0 \tag{22}$$

Optimal values $(\mathcal{P}_c^{\beta it})$ and $(\mathcal{D}_c^{\beta it})$ coincide and, therefore, the fourth addend of the left hand side of (14) can be replaced by $\Gamma_c^i\zeta_{it}^c + \sum_{j\in P_c^a\cup P_c^n} \pi_{jit}^c$ with the following variables and constraints added to the model:

$$\zeta_{it}^c + \pi_{jit}^c \geq \hat{r}_{jt}x_{ji}, \qquad \forall i \in I, j \in P_c^a \cup P_c^n, t \in T$$

$$\zeta_{it}^c \geq 0, \qquad \forall i \in I, t \in T$$

[1] Due to the structure of the model, which aims at considering the worst case, at most one of variables z_{ji}^t is fractional in any optimal solution, if Γ_c^i is not integer, and represents p_c^{it}.

$$\pi^c_{jit} \geq 0, \qquad \forall i \in I, j \in P^a_c \cup P^n_c, t \in T$$

The same idea is applied to $\beta^{it}_{pc}(\xi^*, \Gamma^i_{pc}, t)$ and $\beta^{it}_{nc}(\chi^*, \Gamma^i_{nc}, t)$. The overall robust model guarantees that the optimal solution is feasible for any three subsets of patients with cardinality Γ^i_c, Γ^i_{pc} and Γ^i_{nc}. As also the worst subset is considered among all of the possible subsets, the solution can face the worst scenario.

4. The contribution

In this report, we extend the above mentioned model in two directions:

1. *Multilevel Model*
   The standard formulation of the cardinality-constrained approach considers only two values of demand for each patient and each period of the planning horizon, and this may produce a too conservative solution. In order to overcome this limitation, we propose a new cardinality-constrained model in which several levels of demands are considered. This model is able to produce solutions that are still robust, but less conservative, and therefore cheaper. In this report, we formalize the new model and we describe how to obtain the levels according to the available stochastic information on patients' demands.

2. *Fairness Model*
   Besides the minimization of nurses' overtimes, another important objective pursued by planners is to obtain a fair workload among nurses. According to this, we propose a new objective function for the above mentioned problem, i.e., the fairness of the nurses' utilization. In this report, we give some remarks on the interaction between the fairness objective function and the cardinality-constrained approach, and we analyze the outcomes of the model on a toy example.

5. Multilevel model

For the multilevel model, the cardinality-constrained approach is applied with a different theoretical approach. The original approach does not aim at replicating a realistic demand distribution; on the contrary, it creates a scenario which is likely to be worse than the real executions. Indeed, the approach guarantees that the developed solutions are feasible even with respect to the worst case scenario, although it is very unlikely to occur, thus providing over-conservative solutions. On the contrary, the aim of the multilevel model is to fit the real situation and to optimize with respect to a realistic and not too conservative scenario. Indeed, we want to best fit the possible realizations of the demands, which are generated according to their probability density functions. For this purpose, differently from the standard formulation in which the stochastic demands r_{jt} are characterized only by two values, i.e., the expected value $\bar{r}_{jt}$ and the maximum one $\bar{r}_{jt} + \hat{r}_{jt}$, we introduce a finer differentiation of the demand levels. Starting from the probability density function of each r_{jt} (which is required for solving the multilevel model), we divide the support of the distribution in H intervals. Each interval h (with $h = 1, ..., H$) is comprised between r^{h-1}_{jt} and r^h_{jt}. Obviously, r^0_{jt} corresponds to the minimum value of r_{jt} and r^H_{jt} to its maximum one (Figure 1). Values of each level may occur with probability H^{-1}. We remark that no assumptions have to be introduced on the probability density functions for implementing the division in H equally probable parts, and both continuous and discrete densities can be used. For instance, histograms representing the frequency of different values can be used as discrete

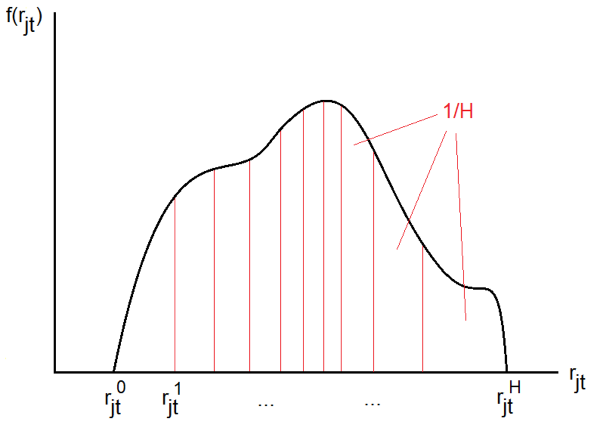

Figure 1: Example of division of the probability density $f(r_{jt})$ in H levels.

density. Alternatively, such densities can be obtained according to specific patient stochastic models, as the one described in [21]. The same division is adopted for all demands r_{jt}, whereas values r_{jt}^h depend on the specific patient demand distribution, and allow taking into account a specific density function $f(r_{jt})$ for each patient j and time slot t.

In the proposed multilevel model, we keep objective function (1). Thus, the formulation of the model before including the robustness is the same. Indeed, the same assignment variables (i.e., decision variables x_{ji}, ξ_{ji}^t and χ_{ji}^t, and parameters $\tilde{x}_{ji}$) and the same overtime model (levels L_i with workload amount w_{it}^l and parameters Δ_i^l and c_l) are adopted. Also in this case, thresholds Δ_i^l may be different from nurse to nurse, while costs c_l are the same and $c_l < c_{l+1}$. Parameters Δ_i^l are defined in such a way that $\sum_{l \in L_i} \Delta_i^l = v_i \; \forall i$. Finally, the same reassignment binary variables y_j^t are used. In this case, we deal with uncertainty by modifying the deterministic formulation with the multilevel version of the cardinality-constrained approach [23]. We introduce $H-1$ integer cardinality parameters Γ_h^{it} (with $h = 2, ..., H$) for each nurse $i \in I$ and time slot $t \in T$, such that $\sum_{h=2}^{H} \Gamma_h^{it} \leq N_{it}$, where N_{it} is the number of uncertain parameters in the considered constraint. We also introduce $H-1$ disjoint sets S_h^{it} (with $h = 2, ..., H$) for each nurse $i \in I$ and each time slot $t \in T$. Each S_h^{it} is the subset of patients assigned to i, whose demand charged to nurse i in time slot t is in interval h of the corresponding density function, i.e., comprised between r_{jt}^{h-1} and r_{jt}^h. For the sake of simplicity, it is assumed that each interval h refers to the demand r_{jt}^h, i.e., to the right border of the interval. Cardinality is constrained similarly than in [2], as at most Γ_h^{it} patients are assumed to belong to subset S_h^{it} for each $i \in I$, $t \in T$ and $h = 2, ..., H$. With respect to the robust model in Section 3.1, we extend the dependency over time giving the possibility to consider a different cardinality Γ_h^{it} for each time slot t. Moreover, we do not create separate sets for the different continuity of care requirements. We remark that, differently from the standard cardinality-constrained approach, parameters r_{jt} not belonging to any subset S_h^{it} assume their minimum possible value r_{jt}^1 and not their nominal value $\bar{r}_{jt}$. Let $\mathcal{P}_{it} = \left\{S_1^{it}, \ldots, S_H^{it}\right\}$ be a partition of the patients assigned to nurse i at time slot t, with the property that $|S_h^{it}| = \Gamma_h^{it}$ for $h = 2, \ldots, H$. Let $\delta_{jt}^h = r_{jt}^h - r_{jt}^1$ for each $j \in P, t \in T, h = 2, \ldots, H$.

Given this configuration, non-robust constraints (5) are modified as follows.

$$
\begin{aligned}
&\sum_{j \in P_c^a \cup P_c^n} r_{jt}^1 x_{ji} + \sum_{j \in P_{pc}^a \cup P_{pc}^n} r_{jt}^1 \xi_{ji}^t + \sum_{j \in P_{nc}} r_{jt}^1 \chi_{ji}^t + \\
&+ \max_{\{S_1^{it},\dots,S_H^{it}\} \in \mathcal{P}_{it}} \left\{ \sum_{h=2}^{H} \left[\sum_{j \in S_h^{it} \cap (P_c^a \cup P_c^n)} \delta_{jt}^h x_{ji} + \right.\right. \qquad (23) \\
&\left.\left. + \sum_{j \in S_h^{it} \cap (P_{pc}^a \cup P_{pc}^n)} \delta_{jt}^h \xi_{ji}^t + \sum_{j \in S_h^{it} \cap P_{nc}} \delta_{jt}^h \chi_{ji}^t \right] \right\} \le v_i + \sum_{l \in L_i} w_{it}^l \quad \forall i \in I, t \in T
\end{aligned}
$$

As required by the robust approach, let us denote with $\beta^{it}(x^*, \xi^*, \chi^*, \Gamma_h^{it})$ the maximum in (23), which is associated to a given solution $\{x^*, \xi^*, \chi^*\}$. It is computed through the following linear programming model $\mathcal{P}^{\beta it}$, where variables z_{ji}^{th} are equal to 1 if patient j belongs to subset S_h^{it}, and 0 otherwise:

$$
\begin{aligned}
(\mathcal{P}^{\beta it}) \quad = \quad & \max \left\{ \sum_{h=2}^{H} \left[\sum_{j \in (P_c^a \cup P_c^n)} \delta_{jt}^h x_{ji}^* z_{ji}^{th} + \right.\right. \\
& \left.\left. + \sum_{j \in (P_{pc}^a \cup P_{pc}^n)} \delta_{jt}^h \xi_{ji}^{t*} z_{ji}^{th} + \sum_{j \in P_{nc}} \delta_{jt}^h \chi_{ji}^{t*} z_{ji}^{th} \right] \right\} \qquad (24)
\end{aligned}
$$

s.t.

$$
\sum_{j \in P} z_{ji}^{th} \le \Gamma_h^{it}, \qquad \forall h = 2, ..., H \qquad (25)
$$

$$
\sum_{h=1}^{H} z_{ji}^{th} = 1, \qquad \forall j \in P \qquad (26)
$$

$$
z_{ji}^{th} \ge 0, \qquad \forall j \in P, i \in I, h \in H, t \in T \qquad (27)
$$

Constraints (25) guarantee that at most Γ_h^{it} patients are assigned to S_h^{it} and constraints (26) ensure that each patient is assigned to at most one set. If we aim at replicating the demand distributions of patients and generate values which refer to a possible real scenario, each Γ_h^{it} must represent the number of the total number of patients assigned to i in interval t divided by H:

$$
\Gamma_h^{it} = \frac{1}{H} \left(\sum_{j \in (P_c^a \cup P_c^n)} x_{ji}^* + \sum_{j \in (P_{pc}^a \cup P_{pc}^n)} \xi_{ji}^{*t} + \sum_{j \in P_{nc}} \chi_{ji}^{t*} \right) \quad \forall h = 2, ..., H \qquad (28)
$$

However, the cardinality-constrained approach requires the linearity of $\mathcal{P}^{\beta it}$ to define the corresponding dual problem $\mathcal{D}$, while the presence of x_{ji}^*, ξ_{ji}^{*t} and χ_{ji}^{t*} in Γ_h^{it} makes the problem nonlinear. In addition, (28) does not guarantee that Γ_k^{it} is integer. We preserve the linearity by considering an approximate value $\widehat{\Gamma}_h^{it}$ for Γ_h^{it}. For this purpose, the terms in parentheses in (28) are replaced by an estimation of the number of patients assigned to i, which is also maintained constant over t. This is computed as the sum of the number of patients in $P_c^a \cup P_{pc}^a$ already assigned to i and the new possible assignments of patients in $P_c^n \cup P_{pc}^n \cup P_{nc}$. The last term is approximated with the upper integer of the number patients in $P_c^n \cup P_{pc}^n \cup P_{nc}$, who are in the same district of i, scaled by the available working time of i normalized with respect to the overall available working time of the district:

$$\widehat{\Gamma}_h^{it} = \frac{1}{H}\left(\sum_{j\in(P_c^a\cup P_{pc}^a)} \tilde{x}_{ji} + \left\lceil \frac{v_i}{\sum_{k\in I_i} v_k} \sum_{j\in(P_c^n\cup P_{pc}^n\cup P_{nc})} m_{ij} \right\rceil\right) \tag{29}$$

where I_i is the subset of nurses operating in the same district of i. We can now derive the dual problem adopting variables ζ_{it} and π_{jit} for each interval h:

$$(\mathcal{D}^{\beta it}) = \min\left\{\sum_{j\in P} \pi_{jit} + \sum_{h=2}^{H} \widehat{\Gamma}_h^{it}\zeta_{it}^h\right\} \tag{30}$$

s.t.

$$\begin{aligned}
\zeta_{it}^h + \pi_{jit} &\geq \delta_{jt}^h x_{ji}^*, && \forall j \in (P_c^a \cup P_c^n), h = 2, ..., H && (31)\\
\zeta_{it}^h + \pi_{jit} &\geq \delta_{jt}^h \xi_{ji}^{t*}, && \forall j \in (P_{pc}^a \cup P_{pc}^n), h = 2, ..., H && (32)\\
\zeta_{it}^h + \pi_{jit} &\geq \delta_{jt}^h \chi_{ji}^{t*}, && \forall j \in P_{nc}, h = 2, ..., H && (33)\\
\pi_{jit} &\geq 0, && \forall j \in P && (34)\\
\zeta_{it}^h &\geq 0, && \forall h = 2, ..., H && (35)
\end{aligned}$$

where ζ are the dual variables associated to (25), and π the dual variables associated to (26). Constraints (23) are then replaced by:

$$\begin{aligned}
&\sum_{j\in P_c^a\cup P_c^n} r_{jt}^1 x_{ji} + \sum_{j\in P_{pc}^a\cup P_{pc}^n} r_{jt}^1 \xi_{ji}^t + \sum_{j\in P_{nc}} r_{jt}^1 \chi_{ji}^t + \\
&\qquad + \sum_{h=2}^{H} \Gamma_h^{it}\zeta_{it}^h + \sum_{j\in P} \pi_{jit} \quad \leq \quad v_i + \sum_{l\in L_i} w_{it}^l, \forall i \in I, t \in T
\end{aligned} \tag{36}$$

and then coupled with the constraints of $\mathcal{D}^{\beta it}$.

6. Fairness model

In the fairness model, we maintain the standard approach of the cardinality-constrained model, with only two levels for the uncertain patients' demands. Thus, the amount of working time r_{jt} is assumed to be an uncertain parameter described by only an expected value $\bar{r}_{jt}$ and a maximum value $\bar{r}_{jt} + \hat{r}_{jt}$. Variables $x_{ji}, \xi_{ji}, \chi_{ji}$ and y_{it} have the same meaning and domain described in Sections 3.1 and 5. On the contrary, to take into account the fairness, the entire workload assigned to nurse i in time slot t is now described by a continuous variable w_{it}, including the possible overtime. Thus, the constraints of the deterministic non-robust model are reformulated as follows:

$$\sum_{i\in I} m_{ij} x_{ji} = 1, \qquad \forall j \in P_c^a \cup P_c^n \tag{37}$$

$$\sum_{i \in I} m_{ij} \xi_{ji}^t = 1, \qquad \forall j \in P_{pc}^a \cup P_{pc}^n, t \in T \tag{38}$$

$$\sum_{i \in I} m_{ij} \chi_{ji}^t = 1, \qquad \forall j \in P_{nc}, t \in T \tag{39}$$

$$\sum_{j \in P_c^a \cup P_c^n} r_{jt} x_{ji} + \sum_{j \in {}_{pc}^a \cup P_{pc}^n} r_{jt} \xi_{ji}^t + \\ + \sum_{j \in P_{nc}} r_{jt} \chi_{ji}^t \leq w_{it}, \qquad \forall i \in I, t \in T \tag{40}$$

$$0 \leq w_{it} \leq 2 v_i, \qquad \forall i \in I, t \in T \tag{41}$$

$$x_{ji} \geq \tilde{x}_{ji}, \qquad \forall i \in I, j \in P_c^a \tag{42}$$

$$y_j^t \geq \xi_{ji}^t - \xi_{ji}^{t-1}, \qquad \forall t \in T \setminus \{t_1\}, j \in P_{pc}^a \cup P_{pc}^n, i \in I \tag{43}$$

$$y_j^{t_1} \geq \xi_{ji}^{t_1} - \tilde{x}_{ji}, \qquad \forall j \in P_{pc}^a, i \in I \tag{44}$$

The classical cardinality-constrained formulation is applied in constraints (40), which are modified as follows:

$$\sum_{j \in P_c^a \cup P_c^n} \bar{r}_{jt} x_{ji} + \Gamma_c^i \zeta_{it}^c + \sum_{j \in P_c^a \cup P_c^n} \pi_{jit}^c + \sum_{j \in P_{pc}^a \cup P_{pc}^n} \bar{r}_{jt} \xi_{ji}^t + \Gamma_{pc}^i \zeta_{it}^{pc} + \\ + \sum_{j \in P_{pc}^a \cup P_{pc}^n} \pi_{jit}^{pc} + \sum_{j \in P_{nc}} \bar{r}_{jt} \chi_{ji}^t + \Gamma_{nc}^i \zeta_{it}^{nc} + \sum_{j \in P_{nc}} \pi_{jit}^{nc} \leq w_{it}, \qquad \forall i \in I, t \in T$$

with:

$$\zeta_{it}^c + \pi_{jit}^c \geq \hat{r}_{jt} x_{ji}, \qquad \forall i \in I, j \in P_c^a \cup P_c^n, t \in T \tag{45}$$

$$\zeta_{it}^{pc} + \pi_{jit}^{pc} \geq \hat{r}_{jt} \xi_{ji}^t, \qquad \forall i \in I, j \in P_{pc}^a \cup P_{pc}^n, t \in T \tag{46}$$

$$\zeta_{it}^{nc} + \pi_{jit}^{nc} \geq \hat{r}_{jt} \chi_{ji}^t, \qquad \forall i \in I, j \in P_{nc}, t \in T \tag{47}$$

Two different objective functions related to fairness and overtime costs are compared. In this analysis we neglect the cost of the reassignments. The first objective is the maximization of the nurses' workload fairness, whereas the second one minimizes the overall overtime costs.

Two alternative functions are proposed for the first objective. The first function simply minimizes the maximum utilization of the nurses:

$$\min \quad z \tag{48}$$

with:

$$z \geq \frac{w_{it}}{v_i}, \qquad \forall i \in I, t \in T \tag{49}$$

However, according to this objective, two solutions with the same optimal value are equivalent even if they provide unfair workloads for some nurses who are not the most loaded ones. The second function overcomes this problem, since the objective is the minimization of a lexicographic function, which takes into account the maximum utilization and the maximum difference between highest (z_{max}) and lowest (z_{min}) nurse utilization:

$$\min \{\alpha z + z_{max} - z_{min}\} \tag{50}$$

However, this objective corresponds to the maximization of z_{min}, which is not compatible with the duality required by the cardinality-constrained approach. Maximizing the values of z_{min} may force the variables deriving from the dual formulation to assume a value greater than their minimum one, and such values are not compatible with the duality properties required by the cardinality-constrained approach. This problem is avoided computing the maximum and the minimum utilization with respect to the expected demands, without considering the robustness. This guarantees the proper value of variables ζ_{it}^{c}, ζ_{it}^{pc}, ζ_{it}^{nc}, π_{jit}^{c}, π_{jit}^{pc} and π_{jit}^{nc}:

$$z_{max} \geq \frac{1}{v_i}\left[\sum_{j\in P_c^a\cup P_c^n} \bar{r}_{jt}x_{ji} + \sum_{j\in P_{pc}^a\cup P_{pc}^n} \bar{r}_{jt}\xi_{ji}^t + \sum_{j\in P_{nc}} \bar{r}_{jt}\chi_{ji}^t\right], \forall i\in I, t\in T \tag{51}$$

$$z_{min} \leq \frac{1}{v_i}\left[\sum_{j\in P_c^a\cup P_c^n} \bar{r}_{jt}x_{ji} + \sum_{j\in P_{pc}^a\cup P_{pc}^n} \bar{r}_{jt}\xi_{ji}^t + \sum_{j\in P_{nc}} \bar{r}_{jt}\chi_{ji}^t\right], \forall i\in I, t\in T \tag{52}$$

In this case, we add the following constraints:

$$\begin{aligned} x_{ji} &\leq m_{ij} && \forall i\in I, j\in P_c^a\cup P_c^n \\ \xi_{ji}^t &\leq m_{ij} && \forall i\in I, j\in P_{pc}^a\cup P_{pc}^n, t\in T \\ \chi_{ji}^t &\leq m_{ij} && \forall i\in I, j\in P_{nc}, t\in T \end{aligned}$$

As said above, we also consider a function that minimizes the overall overtime costs (second objective). The set of overtime levels L_i are introduced for each nurse i and a threshold Δ_i^l and a cost c_l are given for each level $l\in L_i$. For each nurse i and level l, the amount of overtime in time slot t is represented by a continuous non negative variable ω_{it}^l. Thus, the objective function is:

$$\min \sum_{i\in I}\sum_{t\in T}\sum_{l\in L_i} c_l\omega_{it}^l \tag{53}$$

with the following constraints added to compute the overtime amount in each level:

$$w_{it} \leq v_i + \sum_{l\in L_i}\omega_{it}^l, \qquad \forall i\in I, t\in T \tag{54}$$

$$\sum_{l\in L_i}\omega_{it}^l \leq 2v_i, \qquad \forall i\in I, t\in T \tag{55}$$

6.1. Toy example

We consider a small test example with 3 nurses, 12 new patients requiring hard continuity of care, 12 new patients requiring partial continuity of care and 9 patients not requiring continuity of care. We consider a time horizon of 8 weeks and we test all the proposed objective functions. We set $\alpha = 10$, when used. Minimizing the overtime costs, the optimal overtime cost is 38.84, whereas z is equal to 1.259 and the difference between the highest and lowest utilization $z_{max} - z_{min}$ is 0.633. Applying the first fairness objective function, the optimal value of z is equal to 1.188, while the overtime cost rises up to 112.79. The difference between the highest and the lowest utilization $z_{max} - z_{min}$ is reduced, although not considered in the objective function, to 0.627. With the second fairness objective function, we obtain an optimal value of z equal to 1.188, while the overtime cost rises up to 120.79. The difference between the highest and the lowest utilization

$z_{max} - z_{min}$, which is now considered in the objective function, is reduced and it is equal to 0.474. Therefore, it seems useful to consider in the objective function a lexicographic combination of maximum utilization and maximum difference. Values seem to show that overtime cost objective function has a positive impact on fairness, while fairness objective function has not a positive impact on overtime costs. Thus, it is worthy taking into account at least a constraint on the overtime cost. For instance, by adding a constraint that limits the cost to be at most twice the optimal one, we obtain the same values of z and $z_{max} - z_{min}$, while reducing the overtime cost to 77.67. Further, by allowing at most an increase of 50%, the optimal solution has an overtime cost of 58.26 and the same values of z and $z_{max} - z_{min}$. An allowed cost increase of 20% provides a solution with the same z and $z_{max} - z_{min}$, and a further reduction of the overtime cost (equal to 46.60). Hence, it seems that there exist several equivalent optimal solutions with the same maximum utilization and that, by using a lexicographic objective function and suitable budget constraints, the most preferable one can be selected.

7. Discussion and conclusions

In this paper we present two robust formulations for two variants of the nurse-to-patient assignment problem on home care, based on the seminal work of [9]. The first newly proposed model incorporates information about the probability distributions of the uncertain parameters associated to patient demands and it can help to reduce the conservatism of the produced solutions. The second proposed model introduces a fairness based objective function which takes into account nurses workload balancing. The behavior of different versions of the second model are compared on a toy example. The proposed formulations seem promising. Future effort will be devoted to apply the proposed formulations to real instances, to evaluate their performance and the impact of different parameters choices.

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