# A Nonparametric Model for Stationary Time Series

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**Abstract**

We present a family of autoregressive models with nonparametric stationary and transition densities, which achieve substantial modelling flexibility while retaining desirable statistical properties for inference. Posterior simulation involves an intractable normalizing constant; we therefore present a latent extension of the model which enables exact inference through a trans-dimensional MCMC method. We argue the capacity of this family of models to capture time homogeneous transition mechanisms, making them a powerful tool for predictive inference even when the process generating the data does not have a stationary density. Numerical illustrations are presented.

**Keywords**: Stationary time series; Mixture of Dirichlet process model; Latent model.

## 1 Introduction

The mixture of Dirichlet process (MDP) model, introduced by Lo [5], is a very popular model, which has benefitted from the advances in simulation techniques, so that the model is now able to cover more complex data structures, such as regression models and time series models [3].

In the context of time series, there is a need for flexible models which can accommodate complex dynamics, observed in real life data. While stationarity is a desirable property, which facilitates estimation of relevant quantities, it is difficult to construct stationary models for which both the transition mechanism and the invariant density are sufficiently flexible. Many attempts have been made, often resulting in a compromise between flexibility and statistical properties (see e.g. [2, 8, 9, 7, 6]).

We propose a model with nonparametric transition and stationary densities, which enjoys the advantages associated with stationarity, while retaining the necessary flexibility for both the transition and stationary densities. We demonstrate how posterior inference via MCMC can be carried out, focusing on the estimation of the transition density, both for stationary and non–stationary data–generating processes. For ease of exposition, we only consider first order time series data and models, but the construction we propose can be adapted for higher order Markov dependence structures.

## 2 The Model

We construct a nonparametric version of the usual autoregressive model, by defining a nonparametric, i.e. infinite, mixture of parametric bivariate densities $K_\theta(y, x)$, for with both marginals $K_\theta(y)$ and $K_\theta(x)$ are the same. We then define the transition density as the conditional density for $y$ given $x$, therefore preserving the stationarity.

The transition mechanism can be expressed as a nonparametric mixture of transition densities with dependent weights,

$$f_P(y|x) = \sum_{j=1}^{\infty} w_j(x)\, K_{\theta_j}(y|x),$$

where

$$w_j(x) = \frac{w_j\, K_{\theta_j}(x)}{\sum_{j'=1}^{\infty} w_{j'}\, K_{\theta_{j'}}(x)}.$$

Clearly, the expression in the denominator, namely

$$f_P(x) = \sum_{j=1}^{\infty} w_j\, K_{\theta_j}(x),$$

constitutes the invariant density for such transition.

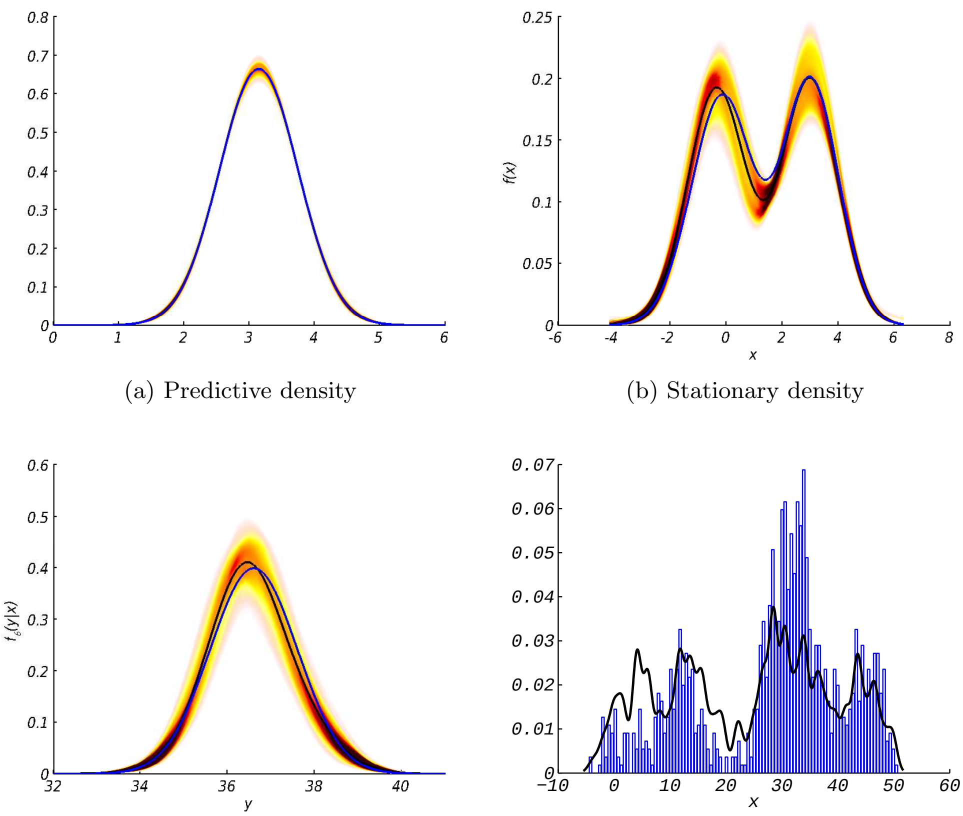
Therefore, both the transition and the stationary densities for the model are defined as nonparametric mixtures.

To our knowledge, the only other fully nonparametric Bayesian model for stationary Markov processes developed so far is due to Martínez-Ovando and Walker [6]. No applications to real data are currently available in the literature, probably due to the complex nature of their model construction.

The model we propose has a simple structure. However, it has been, until now, considered intractable due to the infinite mixture appearing in the denominator of the dependent weight expression. We therefore propose a latent variable extension which enables posterior inference for the model via MCMC, involving slice sampling [4] and a trans-dimensional MCMC method [1]. Future work should include applications to real data.

## 2.1 Illustrations

We present some examples, all of them involving simulated data: form the mixture model itself, a stationary diffusion process, standard Brownian motion and a non-stationary diffusion. They illustrate how our model can be used for transition and invariant density estimation simultaneously, when the stationary density exists; yet remains suitable for transition density estimation, even when the data is not generated by a stationary process.




(a) Predictive density (b) Stationary density

(c) Predictive density (d) Data histogram

Figure 1: Predictive and marginal densities for the mixture model with three mixture components; the true densities (in blue) are accurately captured by posterior inference (heat map). Bottom: Predictive density and histogram for 1000 data points from a Standard Brownian Motion Path; the transition mechanism is recovered by the data, while the marginal provided by the model captures the variability of the data histogram, enabling transition density estimation even in the absence of a true stationary density.

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