Locally adaptive Bayesian covariance regression

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Bayesian Young Statisticians Meeting (BAYSM), Milan June, 5-6, 2013
Paper no. 14

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Abstract

Locally adaptive smoothing to accommodate varying smoothness in a trajectory over time has been well studied, but such approaches have not yet been developed for time-varying covariance matrices to our knowledge. To address this gap, we generalize recently develop methods for Bayesian covariance regression to incorporate random dictionary elements with locally varying smoothness.

Keywords: Bayesian nonparametrics; locally varying smoothness; multivariate time series; nested Gaussian process.

1 Introduction

Multivariate time series data arise in many applied domains, and it is often crucial to obtain a good characterization of how the covariance among the different variables changes over time. Certainly this is the case in financial applications in which covariance can change dramatically during times of financial crisis, revealing different associations among assets and countries than occur in a healthier economic climate. Our focus is on developing models that allow the covariance to vary flexibly over continuous time, and additionally accommodate locally adaptive smoothing of the covariance.

2 The Model

The basic structure of our model recalls the formulation provided by Fox and Dunson [1], which induces a prior on a collection of covariance matrices indexed

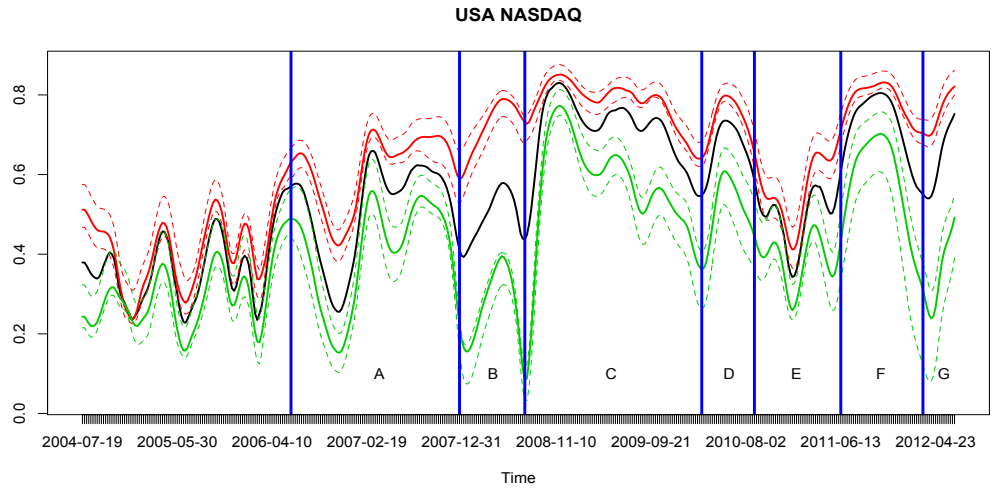

Figure 1: Black line: For USA NASDAQ median of correlations with the other 32 NSI based on posterior mean of $\{\Sigma(t_i)\}_{i=1}^{415}$. Red lines: 25%, 75% (dotted lines) and 50% (solid line) quantiles of correlations between USA NASDAQ and European countries. Green lines: 25%, 75% (dotted lines) and 50% (solid line) quantiles of correlations between USA NASDAQ and the Asian Tigers. Windows in the timeline: (A) burst of U.S. housing bubble, (B) risk of failure of the first U.S. credit agencies (Bear Stearns, Fannie Mae and Freddie Mac), (C) world financial crisis after the Lehman Brothers' bankruptcy, (D) Greek debt crisis, (E) financial reform by Barack Obama and EU efforts to save Greece (the two peaks represent respectively Irish debt crisis and Portugal debt crisis), (F) worsening of European sovereign-debt crisis and rejection of the U.S. budget, (G) crisis of credit institutions in Spain and the growing financial instability of the Eurozone.

by time, through priors for time-dependent loadings matrices in a factor model, characterizing the latter as a sparse combination of a collection of unknown Gaussian process (GP) dictionary functions. Their approach provides a continuous time and highly flexible model that accommodates missing data and scales to large p, but the proposed covariance stochastic process assumes a stationary dependence structure, and hence tends to under-smooth during periods of stability and over-smooth during periods of dramatic change.

In order to provide an higher flexible framework we modify the method of Fox and Dunson to incorporate dictionary functions that are assigned nested Gaussian process (nGP) priors [2]. Using nGP we induce an highly flexible prior on the dictionary functions whose smoothness, explicitly modelled by their derivatives via stochastic differential equations, is expected to be centered on a local instantaneous mean function, which represents an higher-level GP, that induces adaptivity to locally-varying smoothing. Moreover, the possibility to represent such priors through state space models, allows us to develop a fast computational approach via MCMC, with online algorithms also considered.

The results of the simulation study confirm the overall better performance of our approach with respect to our competitors. On the other hand the application

to the multivariate time series of the main national stock market indices before and during the world financial crisis allows to obtain a good characterization of the dynamic dependence structure. The time-varying estimated correlations in Figure 1, show the presence of an evident geo-economic structure in world markets, and a clear international financial contagion effect during the periods of the worsening of the crisis in agreement with other financial theories.

References

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