Efficient Bayesian inference for multivariate factor stochastic volatility models

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Abstract

Multivariate factor stochastic volatility (SV) models are increasingly used for the analysis of multivariate financial and economic time series because they can capture the volatility dynamics by a small number of latent factors. The main advantage of such a model is its parsimony, where all variances and covariances of a time series vector are governed by a low-dimensional common factor with the components following independent SV models. For high dimensional problems of this kind, Bayesian MCMC estimation is a very efficient estimation method, however, it is associated with a considerable computational burden when the number of assets is moderate to large. To overcome this, we avoid the usual forward-filtering backward-sampling (FFBS) algorithm by sampling "all without a loop" (AWOL), consider various reparameterizations such as (partial) non-centering, and apply an ancillarity-sufficiency interweaving strategy (ASIS) for boosting MCMC estimation at an univariate level, w hich can be applied directly to heteroscedasticity estimation for latent variables such as factors. To show the effectiveness of our approach, we apply the model to a vector of daily exchange rate data.

Keywords: Markov chain Monte Carlo; state-space model; dynamic covariance; exchange rate data

1 Introduction

Multivariate factor SV models have recently been applied to important problems in financial econometrics such as asset allocation [1] and asset pricing [4].

They extend standard factor pricing models such as the arbitrage pricing theory and the capital asset pricing model. However, as opposed to SV factor models, standard factor pricing models do not attempt to model the dynamics of the volatilities of the asset returns and usually assume that the covariance matrix $\mathbf{\Sigma}_t \equiv \mathbf{\Sigma}$ is constant. Empirical evidence suggests that multivariate factor SV models are a promising approach for modeling multivariate time-varying volatility, explaining excess asset returns, and generating optimal portfolio strategies.

The model reads

$$\mathbf{y}_t = \mathbf{\Lambda}\mathbf{f}_t + \mathbf{\Sigma}_t^{1/2}\boldsymbol{\epsilon}_t, \qquad \boldsymbol{\epsilon}_t \sim N_m\left(\mathbf{0}, \mathbf{I}_m\right), \tag{1}$$

$$\mathbf{f}_t = \mathbf{V}_t^{1/2}\mathbf{u}_t, \qquad \mathbf{u}_t \sim N_r\left(\mathbf{0}, \mathbf{I}_r\right), \tag{2}$$

where for $t = 1, \ldots, T$, the vector $\mathbf{y}_t = (y_{1t}, \ldots, y_{mt})'$ consists of (potentially demeaned) log-returns of m observed time series, $\mathbf{\Sigma}_t = \mathrm{Diag}(\exp(h_{1t}), \ldots, \exp(h_{mt}))$, $\mathbf{V}_t = \mathrm{Diag}(\exp(h_{m+1,t}), \ldots, \exp(h_{m+r,t}))$, and $\mathbf{\Lambda}$ is an unknown $m \times r$ factor loading matrix with elements Λ_{ij}. The standard assumption is that $\mathbf{f}_t$, $\mathbf{f}_s$, $\boldsymbol{\epsilon}_t$, and $\boldsymbol{\epsilon}_s$ are pairwise independent for all t and s. Both the latent factors and the idiosyncratic shocks are allowed to follow different stochastic volatility processes, i.e.

$$h_{it} = \mu_i + \phi_i(h_{i,t-1} - \mu_i) + \sigma_i\eta_{it}, \qquad \eta_{it} \sim N\left(0, 1\right). \tag{3}$$

In the following, we identify the model by imposing a lower-triangular structure for $\mathbf{\Lambda}$ with unconstrained diagonal elements, and therefore set $\mu_i = 0$ for $i \in \{m+1, \ldots, m+r\}$. Clearly, this introduces an order dependence among the responses and makes the appropriate choice of the first r response variables an important modeling decision.

2 Factor SV Estimation

After fixing $T(m+2r)+mr+4m+3r$, in our application 81 763, starting values for $\boldsymbol{\mu}, \boldsymbol{\phi}, \boldsymbol{\sigma}, \mathbf{h}, \mathbf{f}, \mathbf{\Lambda}$, we repeat the following steps:

a) *Perform $m + r$ univariate SV updates* for $h_{i0}, \ldots, h_{iT}$, ϕ_i, σ_i and m updates for μ_i. We do this by sampling the latent variables AWOL as in [3], thus no FFBS methods are required, there is no need to invert the tridiagonal information matrix of the joint conditional distribution of the latent log-volatilities and computations are fast due to the availability of band back-substitution already implemented in practically all widely used programming languages. Moreover, we employ several variants of ASIS [5] by moving the parameters of interest from the state equation (3) in its centered parameterization to the augmented observation equation (1) or (2) and perform an extra update for these parameters in the noncentered parameterization. Doing so is very cheap in terms of computation – only around 2% extra CPU time is needed – nevertheless has substantial effect on sampling effic iency. Details of efficient univariate SV estimation can be found in [2].

b) *Sample the factor loadings*, constituting m independent r-variate regression problems, from the T-dimensional Gaussian distribution $\mathbf{\Lambda}_{i\cdot}|\mathbf{f}, \mathbf{y}_i, \mathbf{h}_i$. Again we employ ASIS; this time for the diagonal elements Λ_{ii}, which translate nonlinearly to the factor log-volatility levels μ_{m+i}.

c) *Sample the latent factors*, constituting T independent r-variate regression problems, from the m-dimensional Gaussian distribution $\mathbf{f}_t|\mathbf{\Lambda}, \mathbf{y}_{\cdot t}, \mathbf{h}_{\cdot t}$.

Note that all of the above steps can be computed in parallel.

3 Application

We apply a three-factor SV model to EUR exchange rates, quoted indirectly. The data stems from the European Bank's Statistical Data Warehouse and comprises $T = 3140$ observations of 20 currencies ranging from January 3, 2000 to April 4, 2012. Figure 1 shows that individual latent volatilities, displayed in the top graph for the period 2008-2012, exhibit pronounced heteroscedasticity as well as considerable co-movement, thus providing empirical evidence for multivariate modeling through common latent factors. In the bottom graphs, three instantaneous correlation matrices are visualized. It stands out that practically all correlations are positive, but again substantially time-varying. Moreover, some clusters of highly correlated currencies (such as, e.g., the "Asian Tigers") can be spotted, while continental European currencies show little correlation.

References

[1] O. Aguilar, M. West. **Bayesian dynamic factor models and portfolio allocation**. *Journal of Business & Economic Statistics*; 2000; 18(3), pp. 338-357.

[2] G. Kastner, S. Frühwirth-Schnatter. **Ancillarity-sufficiency interweaving strategy (ASIS) for boosting MCMC estimation of stochastic volatility models**. *Computational Statistics & Data Analysis*; 2013; `http://dx.doi.org/10.1016/j.csda.2013.01.002`.

[3] W.J. McCausland, S. Miller, D. Pelletier. **Simulation smoothing for state-space models: A computational efficiency analysis**. *Computational Statistics & Data Analysis*; 2011; 55(1); pp. 199-212.

[4] F. Nardari, J.T. Scruggs. **Bayesian analysis of linear factor models with latent factors, multivariate stochastic volatility, and APT**

pricing restrictions. *Journal of Financial and Quantitative Analysis*; 2007; 42(4), pp. 857-891.

[5] Y. Yu, X.-L. Meng. **To center or not to center: that is not the question — an ancillarity-suffiency interweaving strategy (ASIS) for boosting MCMC efficiency**. *Journal of Computational and Graphical Statistics*; 2011; 20(3); pp. 531-570.

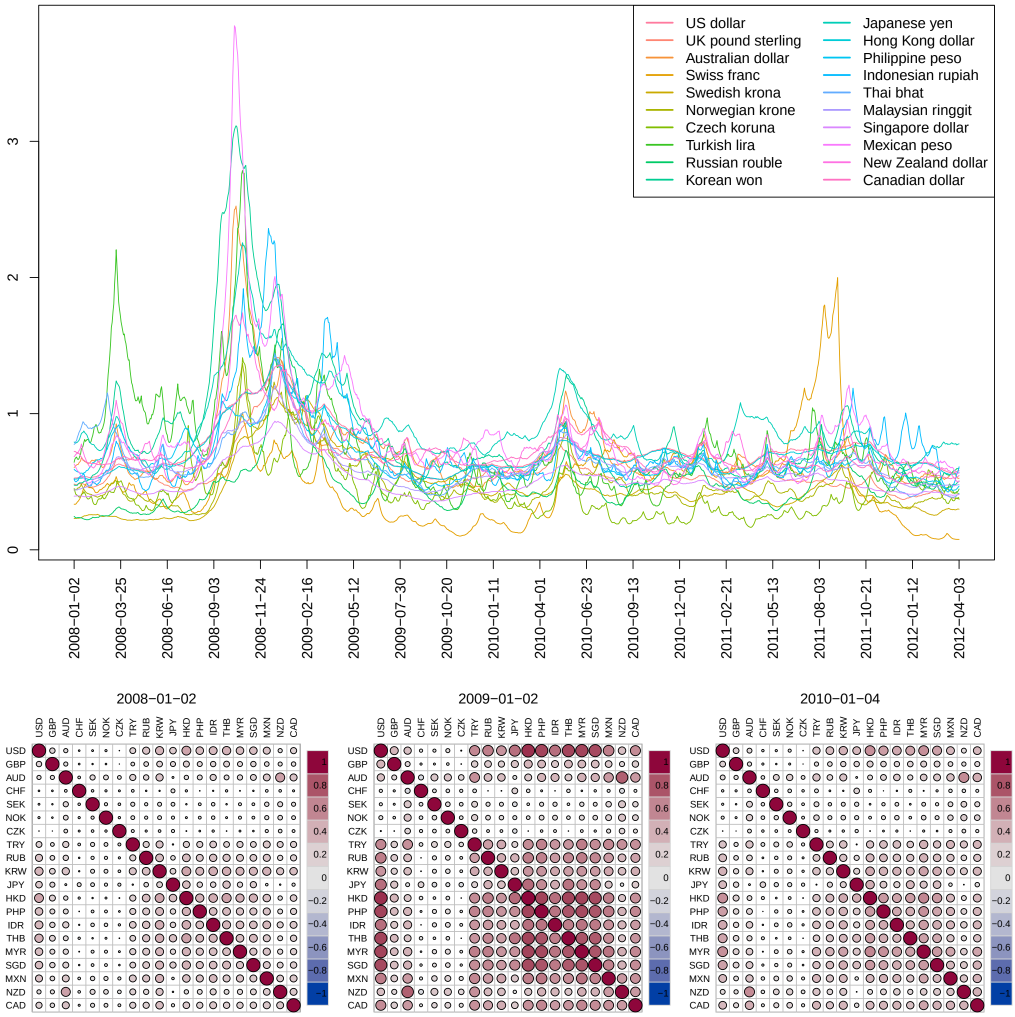

Figure 1: Estimated median univariate volatilities in percent (after 2007) for daily EUR exchange rates (top) and median posterior correlation matrices cov2cor($\mathbf{\Lambda}\mathbf{V}_t\mathbf{\Lambda}' + \mathbf{\Sigma}_t$), exemplified for the first trading days in 2008, 2009 and 2010 (bottom).