Noise Model Selection for Multichannel Diffusion-weighted MRI

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Abstract

We examine the use of various diagnostics for model choice for Multichannel Diffusion-weighted MRI, which is important for inferring the correct tractography, as noise properties can differ between reconstruction techniques and scanners. These are calculated for image data obtained under various different settings of a Philips Achieva 3T scanner. A simulation study carried out which showed these to be reasonably effective at identifying the true model.

Keywords: Diffusion-weighted MRI; ball-and-sticks model; sparsity-inducing priors; information criteria.

1 Introduction

It is well known that the application of multichannel receiver arrays and new image reconstruction techniques, such as parallel imaging, can influence the noise properties in magnetic resonance imaging (MRI) [1]. It has been recently shown that different image reconstructions (which also differ between scanners) of the same multichannel raw data can significantly influence the estimation of fibre orientations from diffusion-weighted MRI [3] and therefore tractography. This is due to their different noise properties and the nature of the DW signal; the signal attenuation is of interest and therefore the signal can be very close to the noise floor. Any changes in the noise properties can directly influence the estimation process.

2 Background

2.1 Data

The raw data were acquired using a Stejskal-Tanner difussion-weighted (DW) pulse sequence within a single-shot EPI (echo planar imaging) protocol. One b=0 s/mm^2 and 61 DW volumes at b=1000 s/mm^2 were acquired in a Philips Achieva 3T scanner. In-plane spatial resolution was 2x2 mm^2 and slice thickness 2mm. A similar protocol was repeated with the DW volumes being acquired at b=3000 s/mm^2. Both acquisitions were first performed using an 8-channel receiver coil and were repeated using a 32-channel coil. Magnitude images were reconstructed from the raw multi-channel data in two different ways, provided by the vendor, CLEAR On (Con) and CLEAR Off (Coff). For the 8-channel and 32-channel data, respectively these images are composed of 112x112x32 voxels and 112x112x8 voxels, from which a subset of 5914 and 2859 voxels were chosen from the midbody of the corpus callosum.

2.2 Models

We are interested in fitting the following ball-and-sticks model [2] for the diffusion signal for the ith acquisition:

$$S_i = S_0 \left((1 - \sum_{j=1}^{N} f_j) \exp\{-b_i d\} + \sum_{j=1}^{N} f_j \exp\left\{-b_i d \mathbf{g}_i^\top \mathbf{v}_j\right\} \right),$$

where the unknown parameters are: d, the diffusivity; S_0, the signal with no diffusion gradients applied; f_j $(j = 1, \ldots, N)$, the proportion of signal described by fibre direction $\mathbf{v}_j$; (θ_j, ϕ_j) $(j = 1, \ldots, N)$, spherical polar coordinates describing fibre direction $\mathbf{v}_j = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. The b-value and gradient direction associated with the ith acquisition, b_i and $\mathbf{g}_i$, respectively, are known.

This model features a mixture of tensors consisting of N perfectly anisotropic tensors (the 'sticks'), each of which depicts one fibre orientation, and a perfectly isotropic tensor (the 'ball'), which captures the rest diffusion processes.

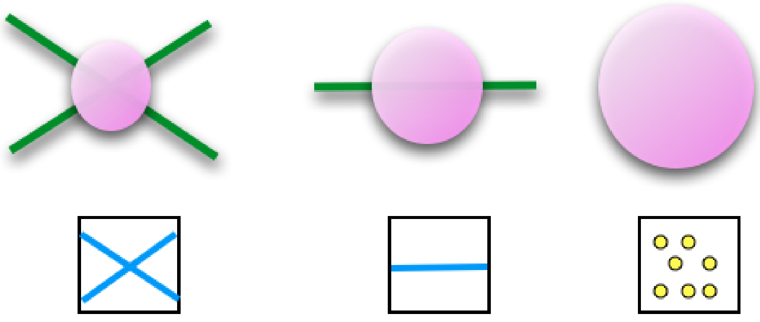

Figure 1: Some examples of ball-and-sticks model

Here we are interested in fitting $N = 3$, in which $N = 1$ and $N = 2$ are nested by setting $f_2 = f_3 = 0$ and $f_3 = 0$, respectively.

We consider five noise models for the reconstructed signal Y_i (with their parameters):

1. **Gaussian/Normal**, $Y_i \sim \mathrm{N}\left(S_i, \tau^{-1}\right)$
2. **Rician**, $Y_i \sim \mathrm{Rice}\left(S_i, \tau\right)$
3. **Noncentral Chi** with **fixed** number of channels ($n = 8/n = 32$), $Y_i \sim \mathrm{NC}\chi_n\left(S_i, \tau\right)$
4. **Noncentral Chi** with **unknown** number of (independent) channels, $Y_i \sim \mathrm{NC}\chi_n\left(S_i, \tau\right)$
5. **Gaussian/Normal** modified by accounting for **higher noise floor**, $Y_i \sim \mathrm{N}\left(S_i, \tau^{-1}\right)$ with

$$S_i = S_0 \left(f_0 + (1 - \sum_{j=0}^{N} f_j) e^{-b_i d} + \sum_{j=1}^{N} f_j \exp\left\{ -b_i d \mathbf{g}_i^\top \mathbf{v}_j \right\} \right)$$

2.3 Methods

We consider three diagnostics for model choice: Akaike Information Criterion (AIC) [4], Bayesian Information Criterion (BIC) [5] and Deviance Information Criterion (DIC) [6]. AIC and BIC are (maximum likelihood) point estimates , and hence can be quick to calculate, though in practice finding the true maximum may be difficult. BIC places a higher penalty on each additional parameter. DIC requires estimation of the likelihood at the mean and the mean of the log-likelihood. In practice this requires a full Bayesian approach using an MCMC algorithm. For speed, we use an MCMC sampler using an adaptive multivariate normal random walk proposal [7] and fit the ball-and-three-sticks model with

Automatic Relevance Determination (ARD) priors [8] proportional to $\frac{1}{f}$ on f_2 and f_3. This sparsity-inducing prior should force these to zero except when they are in truth sufficiently large. This is done to effectively fit three models at once and reduce overfitting.

3 Results summary

It is found that 32 channels, b-values of 3000 and Coff reconstruction tends to favour Non-central Chi noise with degrees of freedom much lower than 32 (indicating correlation between channels), while 8 channels, b-values of 1000 and Con reconstruction favour Gaussian or Rician noise (see table 1).

A simulation study carried out showed ARD priors do a good job in reducing overfitting the number of sticks though generally an ill-fitting noise model leads to overfitting, and hence, incorrect tractography. The diagnostics are seen to identify the true noise model reasonably well.

8ch/b1k/Con		Normal	Rice	NCχ ($n = 8/32$)	NCχ (n unknown)	Mod. Norm.
8ch/b1k/Coff	AIC	49.98	21.98	21.03	2.65	4.35
	BIC	52.65	23.79	22.39	0.61	0.56
	DIC	21.54	24.28	10.57	21.37	22.24
8ch/b1k/Con	AIC	49.02	19.92	23.96	2.79	4.31
	BIC	51.34	21.66	25.75	0.64	0.61
	DIC	21.86	23.03	11.24	20.53	23.33
8ch/b3k/Coff	AIC	6.53	50.59	0.12	31.32	11.45
	BIC	5.83	70.46	0.30	18.26	5.14
	DIC	10.74	39.74	0.34	30.86	18.33
8ch/b3k/Con	AIC	7.73	52.42	0.57	26.90	12.38
	BIC	7.14	70.41	0.90	15.84	5.72
	DIC	10.69	39.75	0.66	28.88	20.02
32ch/b1k/Coff	AIC	50.79	27.95	14.17	2.97	4.13
	BIC	53.17	30.95	13.85	1.36	0.66
	DIC	23.05	24.38	6.44	18.96	27.18
32ch/b1k/Con	AIC	53.17	23.26	16.23	3.50	3.85
	BIC	54.56	26.27	17.10	1.29	0.77
	DIC	22.25	25.11	7.56	18.78	26.30
32ch/b3k/Coff	AIC	8.53	23.57	0.03	52.61	15.25
	BIC	8.43	36.10	0.07	45.33	10.07
	DIC	9.58	22.84	0.07	35.47	32.04
32ch/b3k/Con	AIC	13.92	38.89	0.03	33.89	13.26
	BIC	13.61	52.29	0.07	25.39	8.64
	DIC	13.05	32.00	0.28	25.88	28.79

Table 1: Percentage of voxels for which each noise model yields lowest AIC/BIC/DIC (across all numbers of sticks)

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