Particle Learning Approach to Bayesian Model Selection: An Application from Neurology.

Simon Taylor,[1] Gareth Ridall,[1] Chris Sherlock[1] and Paul Fearnhead[1]

Bayesian Young Statisticians Meeting (BAYSM), Milan, June, 5-6, 2013
Paper no. 39

[1] Department of Mathematics and Statistics, Lancaster University, Lancaster, United Kingdom.
s.taylor2@lancaster.ac.uk
g.ridall@lancaster.ac.uk
c.sherlock@lancaster.ac.uk
p.fearnhead@lancaster.ac.uk

Abstract

An improved method is sought to accurately quantify the number of motor units that operate a working muscle. Measurements of a muscle's contractive potential are obtained by administering a sequence of electrical stimuli, but non-deterministic firing patterns of the motor units impede estimation. We consider a state-space model that assumes a *fixed* number of motor units to describe the hidden processes within the body. Particle learning is applied to estimate the marginal likelihood for a range of models that assumes a different number of motor units. Simulation studies of systems containing up to 8 motor units are very promising.

Keywords: Bayesian Model Selection; Particle Learning; Motor Unit Number Estimation.

1 Introduction

We are interested in accurately quantifying the number of Motor Units (MUs) that supply a working muscle. A MU consists of a single motor neuron and the muscle fibres it governs. An electrical study of a muscle provides insight into the neuromuscular processes by measuring the Compound Muscle Action Potential (CMAP) for a range of stimuli. The ability to partition each CMAP into the contributions from each MU, a Single Motor Unit Potential (SMUP), is central to Motor Unit Number Estimation (MUNE). However, this is complicated by the occurrence of 'alternation' [1], where different MU combinations activate under identical conditions.

2 The Neuromuscular Model

We propose an adaptation to the state-space neuromuscular model [2] that describes the relationship between the applied stimulus, s_t for $t = 1, \ldots, T$, and the corresponding CMAP, y_t, through the hidden biological processes. The state variable is defined to be the firing index vector, $\mathbf{k}_t = (k_{1,t}, \ldots, k_{j,t}, \ldots, k_{u,t})'$, where each element describes a single MU's reaction to the stimulus and the length of this vector, u, denotes the assumed known quantity of MUs within the system. The individual events are assumed to be independent Bernoulli random variables with probability that depends on the administered stimulus via a non-decreasing link function, $F(\cdot;\cdot)$, with parameters specific to the MU, $\boldsymbol{\phi}_j$:

$$k_{j,t}|s_t, \boldsymbol{\phi}_j \quad \sim \quad Bernoulli(\ F(s_t; \boldsymbol{\phi}_j)\ ) \tag{1}$$

Each firing MU generates a SMUP that is assumed to be Gaussian with a unique mean, μ_j, but a common variance, σ^2. Denoting the mean vector of SMUPs as $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_j, \ldots, \mu_u)'$, the recorded CMAP is the sum of the generated SMUPs plus a Gaussian baseline measure that has its own mean, μ_b, and variance, σ_b^2. By using calibration data to approximate σ_b^2, we assume that $\sigma_b^2 \ll \sigma^2$ and define the observation process as follows:

$$y_t|\mathbf{k}_t, \mu_b, \sigma_b^2, \boldsymbol{\mu}, \sigma^2 \quad \sim \quad N(\mu_b + \mathbf{k}_t'\boldsymbol{\mu}, \sigma_b^2 \mathbb{I}_{\{\mathbf{k}_t = \mathbf{0}\}} + \sigma^2 \mathbf{1}'\mathbf{k}_t) \tag{2}$$

3 Methodology

MUNE using the neuromuscular model is assessed by Bayesian model selection; requiring reliable marginal likelihood estimates for a range of proposed model of varying dimension. Consider the marginal predictive factorisation, where each term expresses the probability for a CMAP given the currently available data:

$$\mathbb{P}\left(y_{1:T}|s_{1:T}, u\right) \quad = \quad \mathbb{P}\left(y_1|s_1, u\right) \prod_{t=2}^{T} \mathbb{P}\left(y_t|y_{1:t-1}, s_{1:t}, u\right) \tag{3}$$

Estimates of these terms are obtainable from independent applications of Particle Learning [3] to each considered model. This procedure is an extension of the auxiliary particle filter that constructs the particle set with the 'essential state vector' (ESV), containing the sufficient information necessary for the two stage sequential procedure:

1. Resample the particles with weights proportional to the marginal predictive of y_t with all unknown parameters and state variables marginalised.

2. Propagate the particles either deterministically or by generating appropriate random samples.

The marginal predictive terms are thereby estimated by Monte Carlo integration over the ESV within the procedure before the propagation stages.

4 Discussion

Data from 160 hypothetical neuromuscular systems that contain up to 8 MUs have been simulated. In 139 cases our procedure correctly identified the true number of MUs, according to the *a posteriori* most probable model, and a further 11 cases contained the true scenario within the 95% credible intervals. Figure 1 presents the posterior probabilities for the true model from which the data was generated and it is evident that majority of the misestimation occurred for larger problems. These occurrences are either a result of insufficient information to accurately describe what is happening at the periods of alternation or the difficulty to distinguish between a small SMUP and the noise produced by the other firing MUs.

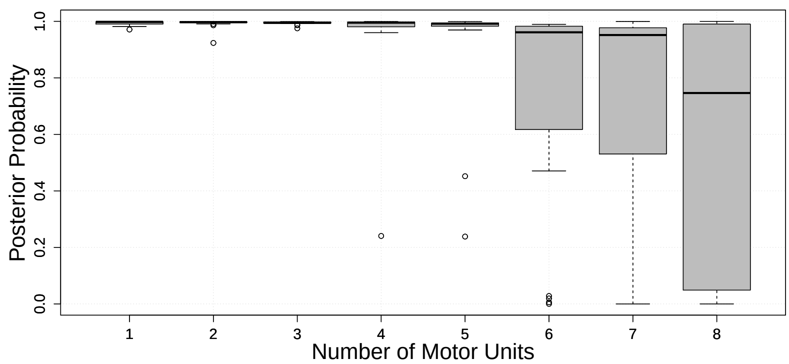

Figure 1: Boxplots of the posterior probabilities for the true number of motor units from which the data was generated.

Our aim is to adapt this procedure to analyse larger neuromuscular systems. However, the event space for $\mathbf{k}_t$ increases exponentially as larger models are considered. Consequently, this substantially increases the computational complexity due to of the need to marginalise all unknowns, parameters and states, within the algorithmic procedure.

References

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